

MODELING OF TURBULENT TRANSPORT TERM OF INTERFACIAL AREA CONCENTRATION IN GAS-LIQUID TWO-PHASE FLOW

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Abstract

A new and rigorous modeling of basic transport equation and constitutive equations of turbulent transport terms of interfacial area concentration was carried out. Based on these local instant formulation, basic transport equation of interfacial area concentration was rigorously formulated in term of spatial correlation functions of characteristic function (local instant volume fraction) and its directional derivative of each phase. In the basic transport equations, interfacial area concentration is transported by averaged interfacial velocity. In the previous models, interfacial velocity is roughly approximated by velocity of each phase. In the present model, interfacial velocity is rigorously formulated in term of spatial correlation functions of characteristic function and velocity of each phase and their directional derivatives. In this new formulation, the averaged interfacial velocity was shown to be correlation functions of fluctuation of velocity and local instant void fraction and their derivatives which reflect the transport of interfacial area concentration due to interaction between interfacial area and turbulence of each phase. Basic conservation equations of spatial correlation functions of characteristic function and velocity of each phase were also derived based on the conservation equations momentum and its fluctuation of each phase. For practical purpose, further modeling of this turbulent transport terms of interfacial area concentration was carried out. As a result, constitutive equations of turbulent diffusion and lateral migration of interfacial area concentration were obtained which can be applied to various flow regime of two-phase flow.

I. Introduction

The two-phase flow phenomenon is playing an important role about safety issues of a nuclear reactor. In order to analyze two-phase flow phenomena, various models such as homogeneous model, slip model, drift flux model and two-fluid model have been proposed. Among these models, the two-fluid model (Ishii (1975), Delhaye (1968)) is considered the most accurate model because this model treats each phase separately considering the phase interactions at gas-liquid interfaces.

More realistic evaluation is attainable especially by taking a multi-dimensional effect into consideration appropriately. That is, it is desirable that many nuclear reactor safety problems are solved by using the two-phase CFD tools, which simulate the complex two-phase flows with a finer space and time resolution than that used in the present two-fluid model codes such as RELAP, TRAC, CATHARE, and ATHLET. There are, however, the following two difficulties to be resolved.

(1) The CPU time drastically increases as the grid becomes fine.

(2) There is no reliable and accurate modeling of the interfacial area for a finer mesh.

The JNES have started the plan shown in Table 1 to prepare the two-phase CFD tool.

Table 1 Plan of preparing the two-phase CFD tool

		2004	2005	2006	2007	2008	2009	2010	2011
Selection of a basis from existing CFD codes ⁽¹⁾		Comparison calculation 1 (steady)		Comparison calculation 2 (transient)					
		<i>Selection of the fully-implicit code (CHAMPAGNE) as a basis⁽²⁾</i>							
The two phase CFD tool	Numerics to relax the drastic increase of CPU time	Coupled solver to solve velocity, volume fraction pressure and/or temperature equations simultaneously							
	Accurate modeling of the interfacial area						New and rigorous modeling of interfacial area concentration transport equation		
Verification and validation of the two phase CFD tool									

(1) ACE-3D, VESUVIUS, CAPE and CHAMPAGNE available in JNES

(2) T. Morii , F. Kasahara, "Preparation of the multidimensional two-phase flow analysis code for nuclear reactor safety regulation in Japan nuclear energy safety organization (JNES)", NURETH-11, Avignon, France, October 2-6, 2005.

In two-fluid model, averaged conservation equations of mass, momentum and energy are formulated for each phase. The conservation equations of each phase are not independent each other and they are strongly coupled through interfacial transfer terms of mass, momentum and energy through gas-liquid interface. Interfacial transfer terms are characteristic terms in two-fluid model and are given in terms of interfacial area concentration (interfacial area per unit volume of two-phase flow) and driving force (Ishii (1975), Ishii and Mishima (1980), Ishii et al. (1982)) as

$$(\text{Interfacial transfer term}) \sim (\text{Interfacial area concentration}) \times (\text{Driving force}) \quad (1)$$

Therefore, the accurate knowledge of interfacial area concentration is quite essential to the accuracy of the prediction based on two-fluid model and a lot of experimental and analytical studies have been made on interfacial area concentration. In conventional codes based on two-fluid model, interfacial area concentration is given in constitutive equations in terms of Weber number of bubbles or droplets depending upon flow regime of two-phase flow (Ransom et al. (1985), Liles et al. (1984)). However, recently, more accurate and multidimensional predictions of two-phase flows are needed for advanced design of nuclear reactors. To meet such needs for improved prediction, it becomes necessary to give interfacial area concentration itself by solving the transport equation. In view of above, recently, intensive researches have been carried out on the models, analysis and experiments of interfacial area transport throughout the world. The extensive review on such researches was recently carried out by Hibiki and Ishii (2009) and Kataoka (2010).

Formulation and modeling of basic transport equation of interfacial area concentration and constitutive equations of the transport equation have been carried out by various researchers. These basic transport equation and constitutive equations were applied to CFD codes of two-phase flow and predictions of detailed behavior of two-phase flow were carried out. In these models, source terms of interfacial area transport due to the break up and coalescence of bubbles were well modeled in flow regimes of bubbly and bubbly-to-slug transition. However, constitutive equations of turbulent transport of interfacial area concentration in various flow regimes have not been modeled nor developed yet.

In the present paper, a new and rigorous modeling of basic transport equation and constitutive equations of turbulent transport terms of interfacial area concentration was carried out. In this new formulation, the averaged interfacial velocity was shown to be correlation functions of fluctuation of velocity and local instant void fraction and their derivatives which reflect the transport of interfacial area concentration due to interaction between interfacial area and turbulence of each phase. Basic conservation equations of spatial correlation functions of characteristic function and velocity of each phase were also derived based on the conservation equations momentum and its fluctuation of each phase. For practical purpose, further modeling of this turbulent transport terms of interfacial area concentration was carried out.

II. Local Instant Formulation of Interfacial Area Concentration and Its Transport Equation

Interfacial area concentration is defined as interfacial area per unit volume of two-phase flow. Therefore, the term “interfacial area concentration” is usually used in the meaning of averaged value and denoted by $\overline{a_i}$. For example, one considers the interfacial area concentration in bubbly flow as shown in Fig.1. The interfacial area concentration is given by

$$\overline{a_i} = \frac{A_i}{V} \quad (2)$$

For simplicity, bubbles are spherical of which diameter is d_b , interfacial area concentration is given by

$$\overline{a_i} = \frac{\pi N d_b^2}{V} = \frac{6\alpha}{d_b} \quad (3)$$

Here, N is number of bubbles in volume V , and α is void fraction (volumetric fraction of bubbles in volume V).

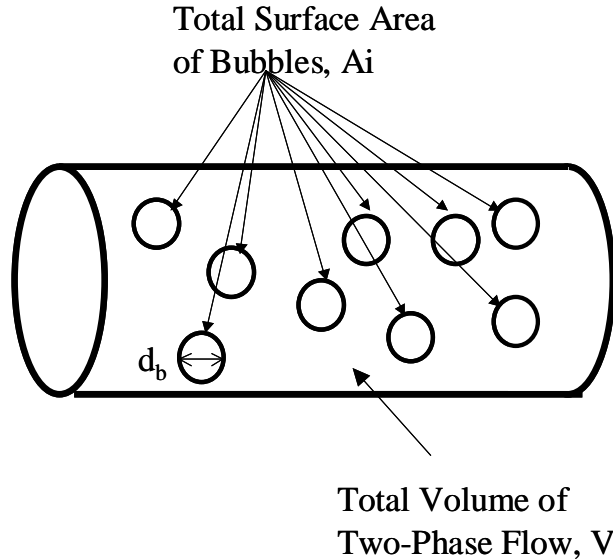


Fig.1 Interfacial area in bubbly flow

The transport equation of interfacial area concentration is given in averaged form in terms

of averaged interfacial area concentration, \bar{a}_i . However, for the rigorous derivation of the transport equation, it is desirable to formulate interfacial area concentration and its transport equation in local instant form. Kataoka et al. (1986), Kataoka (1986) and Morel (2007) derived the local instant formulation of interfacial area concentration as follows.

As Shown in Fig.2, interface of gas and liquid is mathematically given by

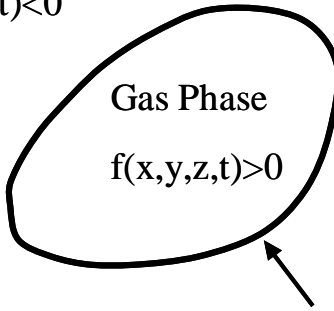
$$f(x,y,z,t)=0 \quad (4)$$

$$f(x,y,z,t)>0 \text{ (gas phase)} \quad (5)$$

$$f(x,y,z,t)<0 \text{ (liquid phase)} \quad (6)$$

Liquid Phase

$$f(x,y,z,t)<0$$



Interface $f(x,y,z,t)=0$

Fig.2 Mathematical representation of interface

Using this function, local instant interfacial area concentration (denoted by a_i) is formulated by

$$a_i = |\text{grad } f(x,y,z,t)| \delta(f(x,y,z,t)) \quad (7)$$

where $\delta(w)$ is the delta function which is defined by

$$\int_{-\infty}^{\infty} g(w) \delta(w - w_0) dw = g(w_0) \quad (8)$$

where $g(w)$ is an arbitrary continuous function..

In relation to local instant interfacial area concentration, characteristic function of each phase (denoted by ϕ_k) is defined by

$$\phi_G = h(f(x,y,z,t)) \text{ (gas phase)} \quad (9)$$

$$\phi_L = 1 - h(f(x,y,z,t)) \text{ (liquid phase)} \quad (10)$$

where suffixes G and L denote gas and liquid phase respectively. ϕ_k is the local instant void fraction of each phase and takes the value of unity when phase k exists and takes the value of zero when phase k doesn't exist. Here, $h(w)$ is Heaviside function which is defined by

$$\begin{aligned} h(w) &= 1 & (w > 0) \\ &= 0 & (w < 0) \end{aligned} \quad (11)$$

Heaviside function and the delta function are related by

$$\delta(w) = \frac{dh(w)}{dw} \quad (12)$$

Using above equations, the derivatives of characteristic function are related to interfacial area concentration as follows.

$$\text{grad } \phi_k = -\mathbf{n}_{ki} a_i \quad (k = G, L) \quad (13)$$

$$\frac{\partial \phi_k}{\partial t} = \mathbf{v}_i \cdot \mathbf{n}_{ki} a_i \quad (k = G, L) \quad (14)$$

Here, \mathbf{n}_{ki} is unit normal outward vector of phase k as shown in Fig.3 and \mathbf{v}_i is the velocity of interface.

Using above-mentioned relations, it is shown that local instant interfacial area concentration is given in term of correlation function of characteristic function as (Kataoka (2008))

$$a_i = -\frac{1}{2\pi} \int_0^{2\pi} \int_0^\pi \left\{ \frac{\partial}{\partial \mathbf{r}} \phi_k(\mathbf{x}) - 2\phi_k(\mathbf{x} + \mathbf{r}) \frac{\partial}{\partial \mathbf{r}} \phi_k(\mathbf{x}) \right\} \sin \theta d\theta d\eta \quad (15)$$

Here, $\frac{\partial}{\partial \mathbf{r}}$ is directional differentiation of characteristic function $\phi_k(\mathbf{x})$ in \mathbf{r} direction.

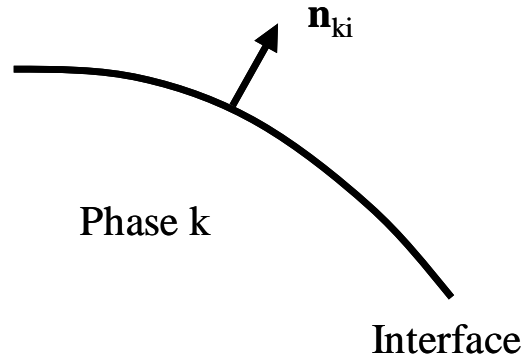


Fig.3 Unit normal outward vector of phase k

Using the local instant formulation of interfacial area concentration described above, Morel (2007) derived the transport equation of averaged interfacial area concentration as given by

$$\frac{\partial \overline{a_i}}{\partial t} + \nabla \cdot \overline{a_i \mathbf{V}_i} = \overline{a_i (\mathbf{V}_i \cdot \mathbf{n}_{Gi}) \nabla \cdot \mathbf{n}_{Gi}} \quad (16)$$

Here, $\overline{\quad}$ denotes averaging and $\overline{\mathbf{V}_i}$ is the averaged velocity of interface which is given by

$$\overline{\mathbf{V}_i} = \overline{a_i (\mathbf{V}_i \cdot \mathbf{n}_{Gi}) \mathbf{n}_{Gi}} / \overline{a_i} \quad (17)$$

Equation (16) is derived by local instant consideration of interfacial area concentration. Therefore, it can be applied to all flow regimes in two-phase flow although constitutive equations of source term (right hand side of Eq.(16)) are needed to solve this equation.

The research group directed by Prof. Ishii in Purdue university derived the transport equation of interfacial area concentration of averaged interfacial area concentration based of the transport equation of number density function of bubbles (Kocamustafaogullari and Ishii (1995), Hibiki and Ishii (2000)). It is given by

$$\frac{\partial \bar{a}_i}{\partial t} + \nabla \cdot \bar{a}_i \mathbf{V}_i = \sum_{j=1}^4 \phi_j + \phi_{ph} \quad (18)$$

Here, the terms in right hand side of Eq.(18) represent the source and sink terms due to bubble coalescence and break up and phase change. The constitutive equations are given by Hibiki and Ishii (2000a, 2000b) and Ishii and Kim (2004) based on detailed mechanistic modeling. Equation (18) is practical transport equation of interfacial area concentration although its applicability is limited to dispersed flow (bubbly flow and droplet flow).

It should be noted that the two equations (16) and (18) are different in nature because these equations are derived based on different models of interfacial area concentration. The averaged interfacial area concentration in Eq.(16) is not exactly same as that in Eq.(18) (Lhuillier et al. (2000)). However, for dispersed two-phase flow where non-homogeneities are not large, the averaged interfacial area concentrations in Eqs.(16) and (18) are approximately equal.

In strict meaning, Eqs. (16) and (18) are “conservation equations of interfacial area concentration”. They are not “transport equations of interfacial area concentration” by themselves. In order to express the transport of interfacial area concentration, it is necessary to formulate the momentum equation of interface considering the various forces acting on the gas-liquid interface. However, at present, most researchers used equations (16) or (10) as “transport equation of interfacial area” assuming interfacial velocity is identical to average velocity of gas phase.

Therefore, in this paper, a new and rigorous definition of velocity of gas-liquid interface is introduced. Based on this definition, the governing equation of velocity of gas-liquid interface is strictly derived

III. Basic Equation of Interfacial Velocity and Transport Equation of Interfacial Area

Interfacial velocity (velocity of gas-liquid interface) is related to the velocity of each phase at interface denoted by \mathbf{v}_{ki} , using following relation.

$$\mathbf{v}_i = \mathbf{v}_{ki} - \mathbf{n}_{ki} \frac{\dot{m}_k}{\rho_k} \quad (19)$$

Here, \dot{m}_k is the mass generation rate of phase k per unit interfacial area due to evaporation or condensation. If there are no phase change or rate of phase change at interface is sufficiently small, the interfacial velocity is equal to the velocity of each phase at interface by

$$\mathbf{V}_i = \mathbf{V}_{Li} = \mathbf{V}_{Gi} \quad (20)$$

In what follows, the most simple case is considered where Eq.(20) is valid.

One considers two spatial locations denoted by \mathbf{x} and $\mathbf{x}+\mathbf{r}$, which are close to each other. There are two cases where gas liquid interface exists between \mathbf{x} and $\mathbf{x}+\mathbf{r}$. They are shown in Figs 4 and 5. Since two locations are very close to each other, it is assumed that only one interface can exist.

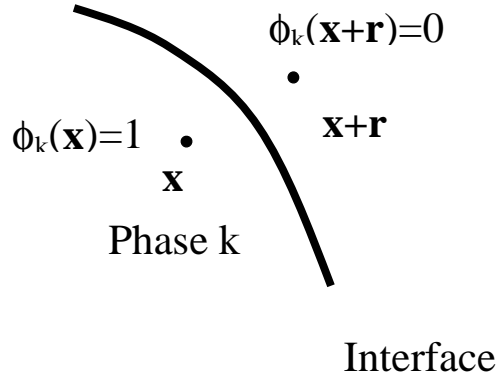


Fig.4 The case where interface exists between \mathbf{x} and $\mathbf{x}+\mathbf{r}$. (\mathbf{x} is in phase k)

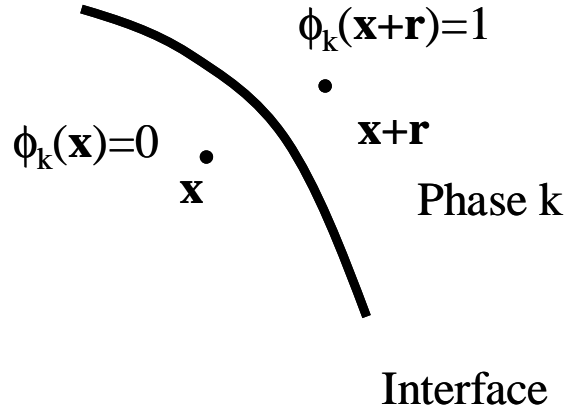


Fig.5 The case where interface exists between \mathbf{x} and $\mathbf{x}+\mathbf{r}$. ($\mathbf{x}+\mathbf{r}$ is in phase k)

In each case, interfacial velocity is approximated by the velocity of phase k at the point where phase k exists. In Fig.4, interfacial velocity is approximated by $\mathbf{v}_k(\mathbf{x})$ and In Fig.5, by $\mathbf{v}_k(\mathbf{x}+\mathbf{r})$. The probabilities where the cases of Fig. 4 and Fig.5 occur are given by

$$\overline{\phi_k(\mathbf{x})\{1-\phi_k(\mathbf{x}+\mathbf{r})\}} \quad , \quad \overline{\{1-\phi_k(\mathbf{x})\}\phi_k(\mathbf{x}+\mathbf{r})}$$

respectively. Then, the probability where gas liquid interface exist between \mathbf{x} and $\mathbf{x}+\mathbf{r}$ is given by

$$\overline{\phi_k(\mathbf{x})\{1-\phi_k(\mathbf{x}+\mathbf{r})\}} + \overline{\{1-\phi_k(\mathbf{x})\}\phi_k(\mathbf{x}+\mathbf{r})}$$

Using above mentioned assumption and probabilities, averaged interfacial velocity is approximated by

$$\frac{\overline{\phi_k(\mathbf{x})\{1-\phi_k(\mathbf{x}+\mathbf{r})\}\mathbf{v}_k(\mathbf{x}) + \{1-\phi_k(\mathbf{x})\}\phi_k(\mathbf{x}+\mathbf{r})\mathbf{v}_k(\mathbf{x}+\mathbf{r})}}{\overline{\phi_k(\mathbf{x})\{1-\phi_k(\mathbf{x}+\mathbf{r})\} + \{1-\phi_k(\mathbf{x})\}\phi_k(\mathbf{x}+\mathbf{r})}}$$

When the displacement, \mathbf{r} , is sufficiently small, averaged interfacial velocity, $\overline{\mathbf{V}}_i$ is given by

$$\overline{\mathbf{V}}_i = \lim_{r \rightarrow 0} \frac{\overline{\phi_k(\mathbf{x})\{1-\phi_k(\mathbf{x}+\mathbf{r})\}\mathbf{v}_k(\mathbf{x}) + \{1-\phi_k(\mathbf{x})\}\phi_k(\mathbf{x}+\mathbf{r})\mathbf{v}_k(\mathbf{x}+\mathbf{r})}}{\overline{\phi_k(\mathbf{x})\{1-\phi_k(\mathbf{x}+\mathbf{r})\} + \{1-\phi_k(\mathbf{x})\}\phi_k(\mathbf{x}+\mathbf{r})}} \quad (21)$$

On the other hand, interfacial area concentration is related to the probability where gas liquid interface exist between \mathbf{x} and $\mathbf{x}+\mathbf{r}$ by following equation (Kataoka et al. (2007), Kataoka and Serizawa (1990)).

$$\overline{a_i} |\cos \theta| = \lim_{|r| \rightarrow 0} \frac{\overline{\phi_k(\mathbf{x})\{1-\phi_k(\mathbf{x}+\mathbf{r})\} + \{1-\phi_k(\mathbf{x})\}\phi_k(\mathbf{x}+\mathbf{r})}}{|r|} \quad (22)$$

Here, θ is the angle between outward normal vector at interface and displacement vector, \mathbf{r} . Using Eq.(22), Eq.(21) can be written by following equation.

$$\overline{\mathbf{V}}_i \overline{a_i} |\cos \theta| = \lim_{r \rightarrow 0} \frac{\overline{\phi_k(\mathbf{x})\{1-\phi_k(\mathbf{x}+\mathbf{r})\}\mathbf{v}_k(\mathbf{x}) + \{1-\phi_k(\mathbf{x})\}\phi_k(\mathbf{x}+\mathbf{r})\mathbf{v}_k(\mathbf{x}+\mathbf{r})}}{|r|} \quad (23)$$

Integrating Eq.(23) for all direction of \mathbf{r} , and averaging, one obtains.

$$\overline{\overline{\mathbf{V}}_i \overline{a_i}} = \frac{1}{2\pi} \int_0^{2\pi} \int_0^\pi \lim_{|r| \rightarrow 0} \frac{\overline{\phi_k(\mathbf{x})\{1-\phi_k(\mathbf{x}+\mathbf{r})\}\mathbf{v}_k(\mathbf{x}) + \{1-\phi_k(\mathbf{x})\}\phi_k(\mathbf{x}+\mathbf{r})\mathbf{v}_k(\mathbf{x}+\mathbf{r})}}{|r|} \sin \theta d\theta d\varphi \quad (24)$$

Similarly, integrating Eq.(22) for all direction of \mathbf{r} , one obtains.

$$\overline{a_i} = \frac{1}{2\pi} \int_0^{2\pi} \int_0^\pi \lim_{|r| \rightarrow 0} \frac{\overline{\phi_k(\mathbf{x})\{1-\phi_k(\mathbf{x}+\mathbf{r})\} + \{1-\phi_k(\mathbf{x})\}\phi_k(\mathbf{x}+\mathbf{r})}}{|r|} \sin \theta d\theta d\varphi \quad (25)$$

Using, Eqs.(24) and (25), the difference between averaged interfacial velocity, $\overline{\mathbf{V}}_i$ and average velocity of phase k, $\overline{\mathbf{v}}_k$ is given by

$$\begin{aligned} & \overline{(\overline{\mathbf{V}}_i - \overline{\mathbf{v}}_k) \overline{a_i}} \\ &= \frac{1}{2\pi} \int_0^{2\pi} \int_0^\pi \lim_{|r| \rightarrow 0} \frac{\overline{\phi_k(\mathbf{x})\{1-\phi_k(\mathbf{x}+\mathbf{r})\}\{\mathbf{v}_k(\mathbf{x}) - \mathbf{v}_k(\mathbf{x})\} + \{1-\phi_k(\mathbf{x})\}\phi_k(\mathbf{x}+\mathbf{r})\{\mathbf{v}_k(\mathbf{x}+\mathbf{r}) - \mathbf{v}_k(\mathbf{x})\}}}{|r|} \sin \theta d\theta d\varphi \end{aligned}$$

(26)

Rearranging the term in integration in the right hand side of Eq.(26) one obtains

$$\begin{aligned} & \lim_{|r| \rightarrow 0} \frac{\overline{\overline{\phi_k(\mathbf{x})\{1-\phi_k(\mathbf{x}+\mathbf{r})\}\{\mathbf{v}_k(\mathbf{x})-\mathbf{v}_k(\mathbf{x}+\mathbf{r})\}} + \{1-\phi_k(\mathbf{x})\}\overline{\overline{\phi_k(\mathbf{x}+\mathbf{r})\{\mathbf{v}_k(\mathbf{x}+\mathbf{r})-\mathbf{v}_k(\mathbf{x})\}}}}}{|r|} \\ &= -\lim_{|r| \rightarrow 0} \frac{\overline{\overline{\phi_k'(\mathbf{x})\phi_k(\mathbf{x}+\mathbf{r})\mathbf{v}_k'(\mathbf{x}+\mathbf{r})}} + \overline{\overline{\phi_k(\mathbf{x})\phi_k'(\mathbf{x}+\mathbf{r})\mathbf{v}_k'(\mathbf{x})}}}{|r|} \end{aligned} \quad (27)$$

Here, ϕ_k' and \mathbf{v}_k' are fluctuating terms of local instant volume fraction and velocity of phase k which are given by

$$\mathbf{v}_k' = \mathbf{v}_k - \overline{\mathbf{v}_k} \quad (28)$$

$$\phi_k' = \phi_k - \overline{\phi_k} \quad (29)$$

Equations (26) and (27) indicate that the difference between averaged interfacial velocity, $\overline{\mathbf{V}_i}$ and average velocity of phase k, $\overline{\mathbf{v}_k}$ is given in terms of correlations between fluctuating terms of local instant volume fraction and velocity of phase k which are related to turbulence terms of phase k.

Then, it is important to derive the governing equation of the correlation term given by Eq.(27). In what follows, one derives the governing equation base on the local instant basic equations of mass conservation and momentum conservation of phase k which are given below (Kataoka (1986)). In these conservation equations, tensor representation is used. Einstein abbreviation rule is also applied. When the same suffix appear, summation for that suffix is carried out except for the suffix k denoting gas and liquid phases.

(Mass conservation)

$$\phi_k \frac{\partial v_{k\beta}}{\partial x_\beta} = 0 \quad (30)$$

(Momentum conservation)

$$\phi_k \frac{\partial v_{k\alpha}}{\partial t} + \phi_k \frac{\partial}{\partial x_\beta} (v_{k\alpha} v_{k\beta}) = -\phi_k \frac{1}{\rho_k} \frac{\partial P_k}{\partial x_\alpha} + \phi_k \frac{1}{\rho_k} \frac{\partial \tau_{k\alpha\beta}}{\partial x_\beta} + \phi_k F_{k\alpha} \quad (31)$$

Averaging Eqs.(30) and (31) one obtains averaged conservation equation of mass and momentum conservation of phase k.

(Averaged mass conservation)

$$\frac{\partial \overline{v_{k\beta}}}{\partial x_\beta} = -\frac{1}{\overline{\phi_k}} \overline{v'_{k\beta i} n_{k\beta i} a_i} \quad (32)$$

(Averaged momentum conservation)

$$\begin{aligned} \frac{\partial \overline{v_{k\alpha}}}{\partial t} + \frac{\partial}{\partial x_\beta} (\overline{v_{k\alpha} v_{k\beta}}) = & -\frac{1}{\rho_k} \frac{\partial \overline{P_k}}{\partial x_\alpha} + \frac{1}{\rho_k} \frac{\partial}{\partial x_\beta} (\overline{\tau_{k\alpha\beta} - \rho_k v'_{k\alpha} v'_{k\beta}}) + \overline{F_{k\alpha}} - \frac{\overline{v_{k\alpha}}}{\overline{\phi_k}} \overline{v'_{k\beta i} n_{k\beta i} a_i} \\ & - \frac{1}{\overline{\phi_k}} \frac{1}{\rho_k} \overline{P'_{ki} n_{k\alpha i} a_i} + \frac{1}{\overline{\phi_k}} \frac{1}{\rho_k} \overline{\tau'_{k\alpha\beta i} n_{k\beta i} a_i} + \frac{1}{\overline{\phi_k}} \overline{v'_{k\alpha} v'_{k\beta} n_{k\beta i} a_i} \end{aligned} \quad (33)$$

Subtracting Eqs(32) and (33) from Eqs.(30) and (31), the conservation equations of fluctuating terms are obtained.

(Conservation equation of mass fluctuation)

$$\phi_k \frac{\partial v'_{k\beta}}{\partial x_\beta} = \frac{\phi_k}{\overline{\phi_k}} \overline{v'_{k\beta i} n_{k\beta i} a_i} \quad (34)$$

(Conservation equation of momentum fluctuation)

$$\begin{aligned} \phi_k \frac{\partial v'_{k\alpha}}{\partial t} + \phi_k \frac{\partial}{\partial x_\beta} (v'_{k\alpha} v'_{k\beta} + v'_{k\alpha} \overline{v_{k\beta}} + v'_{k\beta} \overline{v_{k\alpha}}) = & -\phi_k \frac{1}{\rho_k} \frac{\partial P'_k}{\partial x_\alpha} + \phi_k \frac{1}{\rho_k} \frac{\partial}{\partial x_\beta} (\tau'_{k\alpha\beta} + \rho_k \overline{v'_{k\alpha} v'_{k\beta}}) \\ & + \phi_k F'_{k\alpha} + \frac{\phi_k}{\overline{\phi_k}} \overline{v_{k\alpha} v'_{k\beta i} n_{k\beta i} a_i} + \frac{\phi_k}{\overline{\phi_k}} \frac{1}{\rho_k} \overline{P'_{ki} n_{k\alpha i} a_i} - \frac{\phi_k}{\overline{\phi_k}} \frac{1}{\rho_k} \overline{\tau'_{k\alpha\beta i} n_{k\beta i} a_i} - \frac{\phi_k}{\overline{\phi_k}} \overline{v'_{k\alpha} v'_{k\beta} n_{k\beta i} a_i} \end{aligned} \quad (35)$$

Using Eqs(34) and (35), one can derive conservation equation of the correlation term between fluctuating terms of local instant volume fraction and velocity of phase k. In the derivation, following simplified representation of quantities at the location $\mathbf{x}+\mathbf{r}$.

$$\phi_k(\mathbf{x}+\mathbf{r}) = \phi_{kr} \quad (36)$$

$$A_k(\mathbf{x}+\mathbf{r}) = A_{kr} \quad (37)$$

Here, A_k represents the physical quantity of phase k, such as pressure, velocity etc. Multiplying Eq.(35) by $\phi'_k(\mathbf{x}+\mathbf{r}) = \phi'_{kr}$, and averaging, following conservation equation of correlation term of

fluctuations of local instant volume fraction and velocity of phase k.

$$\begin{aligned}
& \frac{\partial}{\partial t} \overline{(\phi_k \phi'_{kr} v'_{k\alpha})} + \frac{\partial}{\partial X_\beta} \overline{(\phi_k \phi'_{kr} v'_{k\alpha} v_{k\beta})} \\
&= -\frac{1}{\rho_k} \overline{\phi_k \phi'_{kr}} \frac{\partial P'_k}{\partial X_\alpha} + \frac{1}{\rho_k} \overline{\phi_k \phi'_{kr}} \frac{\partial}{\partial X_\beta} (\tau'_{k\alpha\beta} + \rho_k \overline{v'_{k\alpha} v'_{k\beta}}) + \overline{\phi_k \phi'_{kr} F'_{k\alpha}} \\
&+ \frac{\overline{\phi_k \phi'_{kr}}}{\phi_k} \frac{1}{\rho_k} \overline{P'_{ki} n_{k\alpha i} a_i} - \frac{\overline{\phi_k \phi'_{kr}}}{\phi_k} \frac{1}{\rho_k} \overline{\tau'_{k\alpha\beta i} n_{k\beta i} a_i} - \frac{\overline{\phi_k \phi'_{kr}}}{\phi_k} \overline{v'_{k\alpha} v'_{k\beta} n_{k\beta i} a_i} \\
&- \frac{\partial}{\partial X_\beta} \overline{(\phi_k \phi'_{kr} v'_{k\alpha} v'_{k\beta})} - \overline{\phi_k \phi'_{kr} v'_{k\beta}} \frac{\partial v_{k\beta}}{\partial X_\beta} + \overline{v'_{k\alpha} (v_{k\beta} + v'_{k\beta})} \phi_k \frac{\partial}{\partial X_\beta} \overline{\phi'_{kr}} \\
&- \overline{v'_{k\alpha} \phi_k} \frac{\partial}{\partial X_\beta} (\overline{\phi_{kr} v'_{k\beta r}} + \overline{\phi'_{kr} v_{k\beta}})
\end{aligned} \tag{38}$$

Similarly, considering Eq.(35) at $\mathbf{x}+\mathbf{r}$ and multiplying by $\phi'_k(\mathbf{x}) = \phi'_k$, and averaging, following conservation equation of correlation term of fluctuations of local instant volume fraction and velocity of phase k.

$$\begin{aligned}
& \frac{\partial}{\partial t} \overline{(\phi'_k \phi_{kr} v'_{k\alpha r})} + \frac{\partial}{\partial X_\beta} \overline{(\phi'_k \phi_{kr} v'_{k\alpha r} v_{k\beta})} \\
&= -\frac{1}{\rho_k} \overline{\phi'_k \phi_{kr}} \frac{\partial P'_{kr}}{\partial X_\alpha} + \frac{1}{\rho_k} \overline{\phi'_k \phi_{kr}} \frac{\partial}{\partial X_\beta} (\tau'_{k\alpha\beta r} + \rho_k \overline{v'_{k\alpha r} v'_{k\beta r}}) + \overline{\phi'_k \phi_{kr} F'_{k\alpha r}} \\
&+ \frac{\overline{\phi'_k \phi_{kr}}}{\phi_{kr}} \frac{1}{\rho_k} \overline{P'_{ki} n_{k\alpha i} a_{ir}} - \frac{\overline{\phi'_k \phi_{kr}}}{\phi_{kr}} \frac{1}{\rho_k} \overline{\tau'_{k\alpha\beta i} n_{k\beta i} a_{ir}} - \frac{\overline{\phi'_k \phi_{kr}}}{\phi_{kr}} \overline{v'_{k\alpha r} v'_{k\beta r} n_{k\beta i} a_{ir}} \\
&- \frac{\partial}{\partial X_\beta} \overline{(\phi'_k \phi_{kr} v'_{k\alpha r} v'_{k\beta r})} - \overline{\phi'_k \phi_{kr} v'_{k\beta r}} \frac{\partial v_{k\beta r}}{\partial X_\beta} + \overline{v'_{k\alpha r} (v_{k\beta r} + v'_{k\beta r})} \phi_{kr} \frac{\partial}{\partial X_\beta} \overline{\phi'_k} \\
&- \overline{v'_{k\alpha r} \phi_{kr}} \frac{\partial}{\partial X_\beta} (\overline{\phi_r v'_{k\beta}} + \overline{\phi'_k v_{k\beta r}})
\end{aligned} \tag{39}$$

From Eqs.(38) and (39), one obtains the conservation equation of the correlation term given by Eq.(27).

$$\begin{aligned}
& \frac{\partial}{\partial t} \left\{ - \lim_{|\mathbf{r}| \rightarrow 0} \frac{1}{|\mathbf{r}|} (\overline{\phi'_k \phi_{kr} v'_{k\alpha r}} + \overline{\phi_k \phi'_{kr} v'_{kr}}) \right\} + \frac{\partial}{\partial x_\beta} \left(- \lim_{|\mathbf{r}| \rightarrow 0} \frac{1}{|\mathbf{r}|} (\overline{\phi'_k \phi_{kr} v'_{k\alpha r}} + \overline{\phi_k \phi'_{kr} v'_{kr}}) \overline{v_{k\beta}} \right) \\
&= \frac{1}{\rho_k} \lim_{|\mathbf{r}| \rightarrow 0} \frac{1}{|\mathbf{r}|} (\overline{\phi'_k \phi_{kr} \frac{\partial P'_{kr}}{\partial x_\alpha}} + \overline{\phi_k \phi'_{kr} \frac{\partial P'_k}{\partial x_\alpha}}) \\
&- \frac{1}{\rho_k} \lim_{|\mathbf{r}| \rightarrow 0} \frac{1}{|\mathbf{r}|} \left\{ \overline{\phi'_k \phi_{kr} \frac{\partial}{\partial x_\beta} (\tau'_{k\alpha\beta r} + \rho_k \overline{v'_{k\alpha r} v'_{k\beta r}})} + \overline{\phi_k \phi'_{kr} \frac{\partial}{\partial x_\beta} (\tau'_{k\alpha\beta} + \rho_k \overline{v'_{k\alpha} v'_{k\beta}})} \right\} \\
&- \lim_{|\mathbf{r}| \rightarrow 0} \frac{1}{|\mathbf{r}|} (\overline{\phi'_k \phi_{kr} F'_{k\alpha r}} + \overline{\phi_k \phi'_{kr} F'_{k\alpha}}) \\
&- \lim_{|\mathbf{r}| \rightarrow 0} \frac{1}{|\mathbf{r}|} \left(\frac{\overline{\phi'_k \phi_{kr}}}{\phi_{kr}} \frac{1}{\rho_k} \overline{P'_{ki} n_{k\alpha i} a_{ir}} + \frac{\overline{\phi_k \phi'_{kr}}}{\phi_k} \frac{1}{\rho_k} \overline{P'_{ki} n_{k\alpha i} a_i} \right) \\
&+ \lim_{|\mathbf{r}| \rightarrow 0} \frac{1}{|\mathbf{r}|} \left(\frac{\overline{\phi'_k \phi_{kr}}}{\phi_{kr}} \frac{1}{\rho_k} \overline{\tau'_{ki} n_{k\alpha i} a_{ir}} + \frac{\overline{\phi_k \phi'_{kr}}}{\phi_k} \frac{1}{\rho_k} \overline{\tau'_{ki} n_{k\alpha i} a_i} \right) \\
&+ \lim_{|\mathbf{r}| \rightarrow 0} \frac{1}{|\mathbf{r}|} \left(\frac{\overline{\phi'_k \phi_{kr}}}{\phi_{kr}} \overline{v'_{k\alpha r} v'_{k\beta r} n_{k\alpha i} a_{ir}} + \frac{\overline{\phi_k \phi'_{kr}}}{\phi_k} \overline{v'_{k\alpha} v'_{k\beta} n_{k\alpha i} a_i} \right) \\
&+ \lim_{|\mathbf{r}| \rightarrow 0} \frac{1}{|\mathbf{r}|} \left\{ \frac{\partial}{\partial x_\beta} (\overline{\phi'_k \phi_{kr} v'_{k\alpha r} v'_{k\beta r}}) + \frac{\partial}{\partial x_\beta} (\overline{\phi_k \phi'_{kr} v'_{kr} v'_{k\beta}}) \right\} + \lim_{|\mathbf{r}| \rightarrow 0} \frac{1}{|\mathbf{r}|} (\overline{\phi'_k \phi_{kr} v'_{k\beta r}} \frac{\partial v_{k\beta r}}{\partial x_\beta} + \overline{\phi_k \phi'_{kr} v'_{k\beta}} \frac{\partial v_{k\beta}}{\partial x_\beta}) \\
&- \lim_{|\mathbf{r}| \rightarrow 0} \frac{1}{|\mathbf{r}|} \left\{ \overline{v'_{k\alpha r} (v_{k\beta r} + v'_{k\beta r})} \phi_{kr} \frac{\partial}{\partial x_\beta} \phi'_k + \{v'_{k\alpha} (v_{k\beta} + v'_{k\beta})\} \phi_k \frac{\partial}{\partial x_\beta} \phi'_{kr} \right\} \\
&+ \lim_{|\mathbf{r}| \rightarrow 0} \frac{1}{|\mathbf{r}|} \left\{ \overline{v'_{k\alpha r} \phi_{kr} \frac{\partial}{\partial x_\beta} (\phi_r v'_{k\beta} + \phi'_k \overline{v_{k\beta r}})} + \overline{v'_{k\alpha} \phi_k \frac{\partial}{\partial x_\beta} (\phi_{kr} v'_{k\beta r} + \phi'_{kr} \overline{v_{k\beta}})} \right\}
\end{aligned} \tag{40}$$

Using Eq.(40) one finally obtains the governing equation of $\overline{\mathbf{v}_i}$ the difference between averaged interfacial velocity, $\overline{\mathbf{V}_i}$ and average velocity of phase k, $\overline{\mathbf{v}_k}$, given by Eqs.(26) and (27). It is given by

$$\begin{aligned}
& \frac{\partial}{\partial t} \{ \overline{(\mathbf{v}_i - \mathbf{v}_k)} \mathbf{a}_i \} + \frac{\partial}{\partial X_\beta} \{ \overline{(\mathbf{v}_i - \mathbf{v}_k)} \mathbf{a}_i v_{k\beta} \} \\
&= \frac{1}{\rho_k} \frac{1}{2\pi} \int_0^{2\pi} \int_0^\pi \lim_{|r| \rightarrow 0} \frac{1}{|\mathbf{r}|} \overline{(\phi'_k \phi_{kr} \frac{\partial P'_{kr}}{\partial X_\alpha} + \phi_k \phi'_{kr} \frac{\partial P'_k}{\partial X_\alpha})} \sin \theta d\theta d\phi \\
&- \frac{1}{\rho_k} \frac{1}{2\pi} \int_0^{2\pi} \int_0^\pi \lim_{|r| \rightarrow 0} \frac{1}{|\mathbf{r}|} \overline{\{ \phi'_k \phi_{kr} \frac{\partial}{\partial X_\beta} (\tau'_{k\alpha\beta r} + \rho_k v'_{k\alpha r} v'_{k\beta r}) + \phi_k \phi'_{kr} \frac{\partial}{\partial X_\beta} (\tau'_{k\alpha\beta} + \rho_k v'_{k\alpha} v'_{k\beta}) \}} \sin \theta d\theta d\phi \\
&- \frac{1}{2\pi} \int_0^{2\pi} \int_0^\pi \lim_{|r| \rightarrow 0} \frac{1}{|\mathbf{r}|} \overline{(\phi'_k \phi_{kr} F'_{k\alpha r} + \phi_k \phi'_{kr} F'_{k\alpha})} \sin \theta d\theta d\phi \\
&- \frac{1}{2\pi} \int_0^\pi \int_0^{2\pi} \lim_{|r| \rightarrow 0} \frac{1}{|\mathbf{r}|} \overline{(\frac{\phi'_k \phi_{kr}}{\phi_{kr}} \frac{1}{\rho_k} P'_{ki} n_{k\alpha i} a_{ir} + \frac{\phi_k \phi'_{kr}}{\phi_k} \frac{1}{\rho_k} P'_{ki} n_{k\alpha i} a_{i})} \sin \theta d\theta d\phi \\
&+ \frac{1}{2\pi} \int_0^\pi \int_0^{2\pi} \lim_{|r| \rightarrow 0} \frac{1}{|\mathbf{r}|} \overline{(\frac{\phi'_k \phi_{kr}}{\phi_{kr}} \frac{1}{\rho_k} \tau'_{ki} n_{k\alpha i} a_{ir} + \frac{\phi_k \phi'_{kr}}{\phi_k} \frac{1}{\rho_k} \tau'_{ki} n_{k\alpha i} a_{i})} \sin \theta d\theta d\phi \\
&+ \frac{1}{2\pi} \int_0^\pi \int_0^{2\pi} \lim_{|r| \rightarrow 0} \frac{1}{|\mathbf{r}|} \overline{(\frac{\phi'_k \phi_{kr}}{\phi_{kr}} v'_{k\alpha r} v'_{k\beta r} n_{k\alpha i} a_{ir} + \frac{\phi_k \phi'_{kr}}{\phi_k} v'_{k\alpha} v'_{k\beta} n_{k\alpha i} a_{i})} \sin \theta d\theta d\phi \\
&+ \frac{1}{2\pi} \int_0^\pi \int_0^{2\pi} \lim_{|r| \rightarrow 0} \frac{1}{|\mathbf{r}|} \overline{\{ \frac{\partial}{\partial X_\beta} (\phi'_k \phi_{kr} v'_{k\alpha r} v'_{k\beta r}) + \frac{\partial}{\partial X_\beta} (\phi_k \phi'_{kr} v'_{k\alpha} v'_{k\beta}) \}} \sin \theta d\theta d\phi \\
&+ \frac{1}{2\pi} \int_0^\pi \int_0^{2\pi} \lim_{|r| \rightarrow 0} \frac{1}{|\mathbf{r}|} \overline{(\phi'_k \phi_{kr} v'_{k\beta r} \frac{\partial v_{k\beta r}}{\partial X_\beta} + \phi_k \phi'_{kr} v'_{k\beta} \frac{\partial v_{k\beta}}{\partial X_\beta})} \sin \theta d\theta d\phi \\
&- \frac{1}{2\pi} \int_0^\pi \int_0^{2\pi} \lim_{|r| \rightarrow 0} \frac{1}{|\mathbf{r}|} \overline{\{ v'_{k\alpha r} (v_{k\beta r} + v'_{k\beta r}) \phi_{kr} \frac{\partial}{\partial X_\beta} \phi'_k + \{ v'_{k\alpha} (v_{k\beta} + v'_{k\beta}) \phi_k \frac{\partial}{\partial X_\beta} \phi'_{kr} \}} \sin \theta d\theta d\phi \\
&+ \frac{1}{2\pi} \int_0^\pi \int_0^{2\pi} \lim_{|r| \rightarrow 0} \frac{1}{|\mathbf{r}|} \overline{\{ v'_{k\alpha r} \phi_{kr} \frac{\partial}{\partial X_\beta} (\phi_r v'_{k\beta} + \phi'_k v_{k\beta r}) + v'_{k\alpha} \phi_k \frac{\partial}{\partial X_\beta} (\phi_{kr} v'_{k\beta r} + \phi'_{kr} v_{k\beta}) \}} \sin \theta d\theta d\phi
\end{aligned} \tag{41}$$

As shown above, the rigorous formulation of governing equation of interfacial velocity is derived. Then, the most strict formulation of transport equations of interfacial area concentration is given by conservation equation of interfacial area concentration (Eq.(16) or Eq.(18)) and conservation equation of interfacial velocity (Eq.(41)). As shown in Eq.(41), the conservation equation of interfacial velocity consists of various correlation terms of fluctuating terms of velocity and local instant volume fractions. These correlation terms represent the turbulent transport of interfacial area, which reflects the interactions between gas liquid interface and turbulence of gas and liquid phases. Equation (41) rigorously represents such turbulence transport terms of interfacial area concentration. Accurate predictions of interfacial area transport can be possible by solving the transport equations derived here.

IV. Constitutive Equations of Turbulent Transport of Interfacial Area

As shown in the previous section, the rigorous formulation of transport equation of interfacial area concentration was derived based on the basic conservation equation of fluctuating terms of local instant volume fraction and velocity of each phase in gas-liquid two-phase flow. They are given by conservation equation of interfacial area concentration (Eq.(16) or Eq(18)) and conservation equation of interfacial velocity (Eq.(41)). However, Eq.(41) consists of complicated correlation terms of fluctuating terms of local instant volume fraction, velocity, pressure and shear stress. The detailed knowledge of these correlation terms is not available. Therefore, solving Eq.(41) together with basic equations of two-fluid model is difficult at present. More detailed analytical and experimental works on turbulence transport terms of interfacial area concentration are necessary for solving practically Eq.(41).

Therefore, here, constitutive correlation of interfacial velocity will be derived based on the conservation equation of interfacial velocity given by Eq.(41). Using such constitutive correlation of interfacial velocity, conservation equation of interfacial area concentration (Eq.(16) or Eq(18)) will represent interfacial area transport more accurately.

From Eqs.(26) and (27), Interfacial velocity is related to averaged velocity of phase k (gas phase or liquid phase) by following equation.

$$\overline{\mathbf{V}_i a_i} = \overline{\mathbf{v}_k a_i} - \frac{1}{2\pi} \int_0^{2\pi} \int_0^\pi \lim_{|\mathbf{r}| \rightarrow 0} \frac{1}{|\mathbf{r}|} (\overline{\phi'_k \phi_{kr} \mathbf{v}'_{kr}} + \overline{\phi_k \phi'_{kr} \mathbf{v}'_k}) \sin \theta d\theta d\varphi \quad (42)$$

When one considers bubbly flow and phase k is assigned to gas phase, Eq.(42) can be rewritten by

$$\overline{\mathbf{V}_i a_i} = \overline{\mathbf{v}_G a_i} - \frac{1}{2\pi} \int_0^{2\pi} \int_0^\pi \lim_{|\mathbf{r}| \rightarrow 0} \frac{1}{|\mathbf{r}|} (\overline{\phi'_G \phi_{Gr} \mathbf{v}'_{Gr}} + \overline{\phi_G \phi'_{Gr} \mathbf{v}'_G}) \sin \theta d\theta d\varphi \quad (43)$$

Here, the term, $\phi'_G / |\mathbf{r}|$ is related to the fluctuating term of interfacial area concentration at the location, \mathbf{x} and the term, $\phi_{Gr} \mathbf{v}'_{Gr}$ is the fluctuating term of gas phase velocity at the location, $\mathbf{x}+\mathbf{r}$. Also, the term, $\phi'_G / |\mathbf{r}|$ is related to the fluctuating term of interfacial area concentration at the location, $\mathbf{x}+\mathbf{r}$ and the term, $\phi_G \mathbf{v}'_G$ is the fluctuating term of gas phase velocity at the location, \mathbf{x} . Then, the correlation term of these fluctuation terms given by

$$-\frac{1}{|\mathbf{r}|} (\overline{\phi'_G \phi_{Gr} \mathbf{v}'_{Gr}} + \overline{\phi_G \phi'_{Gr} \mathbf{v}'_G})$$

is considered to correspond to turbulent transport term due to the turbulent velocity fluctuation. In analogous to the turbulent transport of momentum, energy (temperature) and mass, the correlation term described above is assumed to be proportional to the gradient of interfacial area concentration which is transported by turbulence (diffusion model). Then, one can assume following relation.

$$-\frac{1}{2\pi} \int_0^{2\pi} \int_0^\pi \lim_{|\mathbf{r}'| \rightarrow 0} \frac{1}{|\mathbf{r}'|} (\overline{\phi'_G \phi_{Gr} \mathbf{v}'_{Gr}} + \overline{\phi_G \phi'_{Gr} \mathbf{v}'_G}) \sin \theta d\theta d\phi = -D_{ai} \overline{\text{grad}_i} \quad (44)$$

Here, the coefficient, D_{ai} is considered to correspond to turbulent diffusion coefficient of interfacial area concentration. In analogy to the turbulent transport of momentum, energy (temperature) and mass, this coefficient is assumed to be given by

$$D_{ai} \propto |\mathbf{v}'_G| L \quad (45)$$

Here, L is the length scale of turbulent mixing of gas liquid interface and $|\mathbf{v}'_G|$ is the turbulent velocity of gas phase. In bubbly flow, it is considered that length scale of turbulent mixing of gas liquid interface is proportional to bubble diameter, d_B and length scale of the turbulent velocity of gas phase is proportional to the turbulent velocity of liquid phase. These assumptions were confirmed by experiment and analysis of turbulent diffusion of bubbles in bubbly flow (Kataoka and Serizawa (1991a)). Therefore, turbulent diffusion coefficient of interfacial area concentration is assumed by following equation.

$$D_{ai} = K |\mathbf{v}'_L| d_B = 6K \frac{\alpha}{a_i} |\mathbf{v}'_L| \quad (46)$$

Here, α is the averaged void fraction and $|\mathbf{v}'_L|$ is the turbulent velocity of liquid phase. K is empirical coefficient. For the case of turbulent diffusion of bubble, experimental data were well predicted assuming $K=1/3$ (Kataoka and Serizawa (1991a)). For the case of turbulent diffusion of interfacial area concentration, there are no direct experimental data of turbulent diffusion. However, the diffusion of bubble is closely related to the diffusion of interfacial area (surface area of bubble). Therefore, as first approximation, the value of k for bubble diffusion can be applied to diffusion of interfacial area concentration in bubbly flow. Equations (45) and (46) can be approximately applied to slug flow and churn flow where gas phase is dispersed in liquid phase. However, the length scale of turbulent mixing of gas liquid interface should be appropriately given. For slug flow and churn flow, the length scale of turbulent mixing of gas liquid interfaces is considered to be sufficiently large and order of diameter of flow passage. Therefore, the length scale of turbulent mixing of gas liquid interfaces can be approximated by diameter of flow passage and empirical coefficient must be given by experiments.

Equations (44) is based on the model of turbulent diffusion of interfacial area concentration. In this model, it is assumed that turbulence is isotropic. However, in the practical two-phase flow in the flow passages turbulence is not isotropic and averaged velocities and turbulent velocity have distribution in the radial direction of flow passage. In such non-isotropic turbulence, the correlation terms of turbulent fluctuation of velocity and interfacial area concentration given by Eq.(27) is largely dependent on anisotropy of turbulence field.

Such non-isotropic turbulence is related to the various terms consisting of turbulent stress which appear in the right hand side of Eq.(41). Assuming that turbulent stress of gas phase is proportional to that of liquid phase and turbulence model in single phase flow, turbulent stress is given by

$$\overline{\mathbf{v}'_L \mathbf{v}'_L} = \varepsilon_{LTP} \nabla \overline{\mathbf{v}_L} \quad (47)$$

Here, ε_{LTP} is the turbulent diffusivity of momentum in gas-liquid two-phase flow. For bubbly flow, this turbulent diffusivity is given by various researchers (Sato et al. (1981), Kataoka and Serizawa (1993)). Kataoka and Serizawa (1993) proposed the following correlation.

$$\varepsilon_{LTP} = \frac{1}{3} \alpha d_B v'_L \quad (48)$$

Based on the model of turbulent stress in gas-liquid two-phase flow and Eq.(41), it is assumed that turbulent diffusion of interfacial area concentration due to non-isotropic turbulence is proportional to the velocity gradient of liquid phase. For the diffusion of bubble due to non-isotropic turbulence in bubbly flow in pipe, Kataoka and Serizawa (1991b) proposed the following correlation based on the analysis of radial distributions of void fraction and bubble number density.

$$J_B = K \alpha d_B n_B \frac{\partial \overline{v_L}}{\partial y} \quad (49)$$

Here, J_B is the bubble flux in radial direction and n_B is the number density of bubble. y is radial distance from wall of flow passage. K is empirical correlation and experimental data were well predicted assuming $K=10$. In analogous to Eq.(49), it is assumed that turbulent diffusion of interfacial area concentration due to non-isotropic turbulence is given by following equation.

$$J_{ai} = K \alpha d_B a_i \frac{\partial \overline{v_L}}{\partial y} \quad (50)$$

Here, J_{ai} is the flux of interfacial area concentration in radial direction. Equation (49) can be interpreted as equation of bubble flux due to the lift force due to liquid velocity gradient. Zun (1988) proposed another form of equation given bubble flux which is given by

$$J_B = 0.8 \exp(-2\alpha) \alpha n_B u_R^2 \frac{\partial \overline{v_L}}{\partial y} / g \quad (51)$$

Here, u_R is the relative velocity between gas phase (bubble) and liquid phase in flow direction which is given by $|\overline{\mathbf{v}_G} - \overline{\mathbf{v}_L}|$. Based on Eq.(51), the flux of interfacial area concentration in radial direction, J_{ai} is assumed to be given by

$$J_{ai} = 0.8 \exp(-2\alpha) \alpha a_i u_R^2 \frac{\partial \overline{v_L}}{\partial y} / g \quad (52)$$

As shown above, turbulent diffusion of interfacial area concentration due to non-isotropic turbulence is related to the gradient of averaged velocity of liquid phase and using analogy to the lift force of bubble, Eq.(44) can be written in three dimensional form by

$$-\frac{1}{2\pi} \int_0^{2\pi} \int_0^\pi \lim_{|r| \rightarrow 0} \frac{1}{|r|} (\overline{\phi'_G \phi_{Gr} \mathbf{v}'_{Gr}} + \overline{\phi'_G \phi'_{Gr} \mathbf{v}'_G}) \sin \theta d\theta d\phi = -D_{ai} \overline{\text{grad} a_i} + C \alpha a_i (\overline{\mathbf{v}_G} - \overline{\mathbf{v}_L}) \times \overline{\text{rot}(\mathbf{v}_L)} \quad (53)$$

Empirical coefficient C in the right hand side of Eq.(53) should be determined based on the experimental data of spatial distribution of interfacial area concentration and averaged velocity of each phase. However, at present, there are not sufficient experimental data. Therefore, as first approximation, the value of coefficient C can be given by Eq.(50) or (52).

$$C = Kd_B / u_R \quad (\text{based on Eq.(50)}) \quad (54)$$

$$C = 0.8 \exp(-2\alpha) u_R / g \quad (\text{based on Eq.(52)}) \quad (55)$$

Using Eqs(43) and (53), transport equation of interfacial area concentration Eq.(18) can be given by following equation for gas-liquid two-phase flow where gas phase is dispersed in liquid phase (bubbly flow, slug flow and churn flow).

$$\begin{aligned} \frac{\partial \overline{a_i}}{\partial t} + \overline{\text{div}(a_i \mathbf{v}_G)} = \\ \overline{\text{div}(D_{ai} \text{grad} a_i)} - \overline{\text{div}\{C \alpha a_i (\overline{\mathbf{v}_G} - \overline{\mathbf{v}_L}) \times \overline{\text{rot}(\mathbf{v}_L)}\}} + \frac{2}{3} \frac{\overline{a_i}}{\alpha \rho_G} \left(\overline{\Gamma_G} - \alpha \frac{D\rho_G}{Dt} \right) + \overline{\phi_{CO}} + \overline{\phi_{BK}} \end{aligned} \quad (56)$$

Here, D/Dt denotes material derivative following the gas phase motion and turbulent diffusion coefficient of interfacial area concentration, D_{ai} is given by Eq.(45) or (46). Coefficient of turbulent diffusion of interfacial area concentration due to non-isotropic turbulence, C is given by Eq.(54) and (55). The third term in the right hand side of Eq.(56) is source term of interfacial area concentration due to phase change and density change of gas phase due to pressure change. $\overline{\Gamma_G}$ is the mass generation rate of gas phase per unit volume of two-phase flow due to evaporation. $\overline{\phi_{CO}}$ and $\overline{\phi_{BK}}$ are sink and source term due to bubble coalescence and break up. These source and sink terms are given by Hibiki and Ishii (2000) and Yao and Morel (2004) based on detailed analysis and experiment of interfacial area concentration. More detailed experimental data of interfacial area concentration are needed in order to determine the coefficients in Eq.(56) and to validate source and sink term particularly for slug and churn flow regimes.

For droplet flow where liquid phase is dispersed in gas phase, basic equation of conservation of interfacial area concentration (Eq.(16) or (18)) and basic equation of interfacial velocity (41) are also valid. Since gas phase is continuous phase in droplet flow, Eq.(43) is rewritten by following equation.

$$\overline{\mathbf{V}_i a_i} = \overline{\mathbf{v}_L a_i} - \frac{1}{2\pi} \int_0^{2\pi} \int_0^\pi \lim_{|r| \rightarrow 0} \frac{1}{|r|} (\overline{\phi'_L \phi_{Lr} \mathbf{v}'_{Lr}} + \overline{\phi'_L \phi'_{Lr} \mathbf{v}'_L}) \sin \theta d\theta d\phi \quad (57)$$

For droplet flow, droplets are dispersed phase and transported by the turbulent diffusion due to gas

phase turbulence. The second term in right hand side of Eq.(57) can be formulated similar to the flow regime where gas phase is dispersed in liquid phase (Eq.(56)). It is given by

$$-\frac{1}{2\pi} \int_0^{2\pi} \int_0^\pi \lim_{|r| \rightarrow 0} \frac{1}{|\mathbf{r}|} (\overline{\phi'_L \phi_{Lr} \mathbf{v}'_{Lr}} + \overline{\phi_L \phi'_{Lr} \mathbf{v}'_L}) \sin \theta d\theta d\phi = -D_{ai} \overline{\text{grad} a_i} + C(1-\alpha) \overline{a_i} (\overline{\mathbf{v}_L} - \overline{\mathbf{v}_G}) \times \text{rot}(\overline{\mathbf{v}_G}) \quad (58)$$

Here, turbulent diffusion coefficient of interfacial area concentration is approximated by turbulent diffusion coefficient of droplet (Cousins and Hewitt (1968)) as first approximation. The coefficient C for turbulent diffusion of interfacial area concentration due to non-isotropic turbulence (or lift force term) can be approximated by lift force coefficient of solid sphere as first approximation. Then, transport equation of interfacial area concentration for droplet flow is given by

$$\frac{\partial \overline{a_i}}{\partial t} + \text{div}(\overline{a_i \mathbf{v}_L}) = \text{div}(D_{ai} \overline{\text{grad} a_i}) - \text{div}\{C(1-\alpha) \overline{a_i} (\overline{\mathbf{v}_L} - \overline{\mathbf{v}_G}) \times \text{rot}(\overline{\mathbf{v}_G})\} + \frac{2}{3} \frac{\overline{a_i}}{(1-\alpha)\rho_L} \left(\Gamma_L - (1-\alpha) \frac{D\rho_L}{Dt} \right) + \phi_{CO} + \phi_{BK} \quad (59)$$

Here, D/Dt denotes material derivative following the gas phase motion and ϕ_{CO} and ϕ_{BK} are sink and source terms due to droplet coalescence and break up and Γ_L is the mass generation rate of liquid phase per unit volume of two-phase flow due to condensation.

V. Conclusions

A rigorous formulation of basic transport equations of interfacial area concentration is derived. The basic transport equations consist of conservation equation of interfacial area concentration (Eq.(16) or (18)) and conservation equation of turbulent transport term of interfacial area (Eq.(41)). In the previous model, interfacial velocity is approximated by averaged velocity of gas phase or liquid phase. In the present model, interfacial velocity is formulated by spatial correlation of characteristic function and velocity of each phase (Eq.(21)). Then, the difference between interfacial velocity and averaged velocity of each phase is related to the correlations of fluctuation of velocity and characteristic function which reflect turbulent transport of interfacial area concentration (Eq.(27)). Based on these local instant formulation of gas-liquid two-phase flow, conservation equation of turbulent transport term of interfacial area concentration is derived (Eq.(41)). The basic equation of interfacial area concentration (Eq.(16) or (18)) and conservation equation of turbulent transport term of interfacial area (Eq.(41)) are the most rigorous formulation of the transport equations of interfacial area concentration.

For practical purpose, simplified modeling of turbulent transport terms of interfacial area concentration was carried out. The constitutive equations of turbulent diffusion and lateral migration of interfacial area concentration were obtained which can be applied to various flow regime of two-phase flow. Using these constitutive equations a simplified basic transport equation of interfacial area concentration was derived considering turbulent transport of interfacial area concentration (Eq.(56) and (59)).

More detailed experimental data are needed in order to validate basic equations and constitutive equations of interfacial area concentration transport derived here.

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